

# Separating Verifiable Delay Functions and Time-Lock Puzzles

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Joint work with Hamza Abusalah‡, Karen Azari\* and Chethan Kamath†

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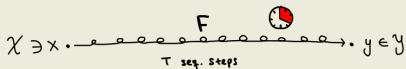
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3. Proof strategy

# Verifiable Delay Functions (VDFs)

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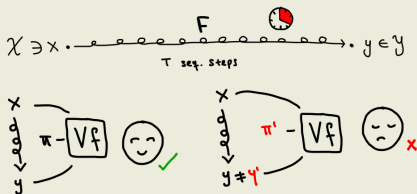


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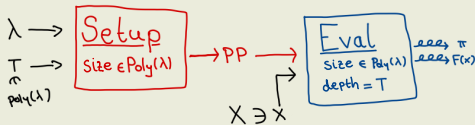


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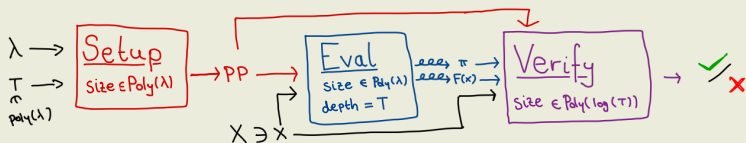


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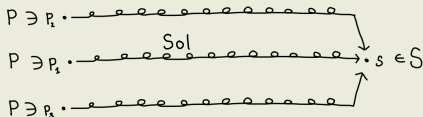
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**Soundness:** given  $(pp, x)$ , no PPT adversary can find  $(y', \pi')$  s.t.  $y' \neq F(x)$  and  $\text{Verify}(pp, x, y', \pi') = 1$ .

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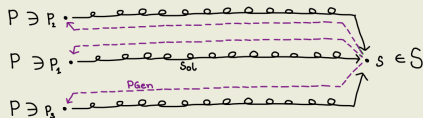
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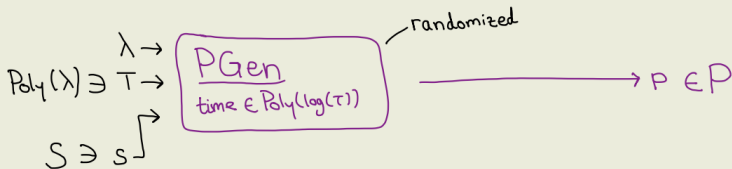
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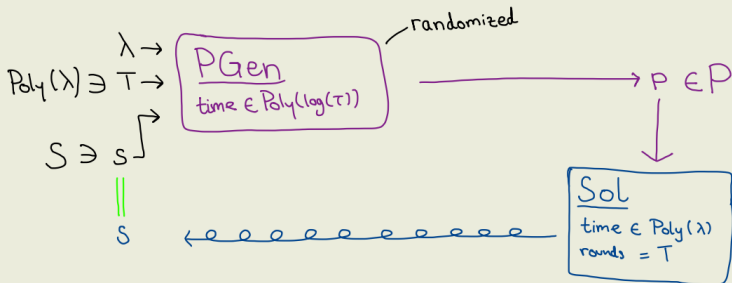
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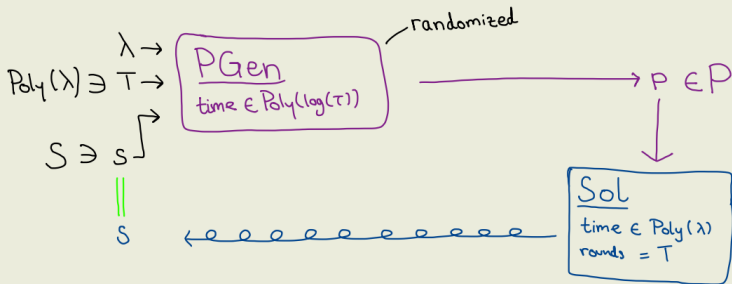
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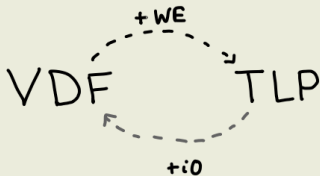
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**Indistinguishability of Puzzles.** For any  $s_0, s_1 \in \mathcal{S}$ ,  
 $\text{PGen}(s_0, T) \approx_{\text{depth} \ll T} \text{PGen}(s_1, T)$

# Known Implications

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**However**, these constructions

- rely on additional assumptions
- are *not* black-box

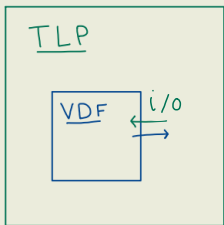
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**Example.** A fully black-box construction  $VDF \rightarrow TLP$  would be...



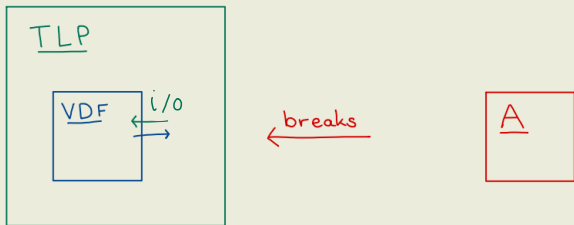
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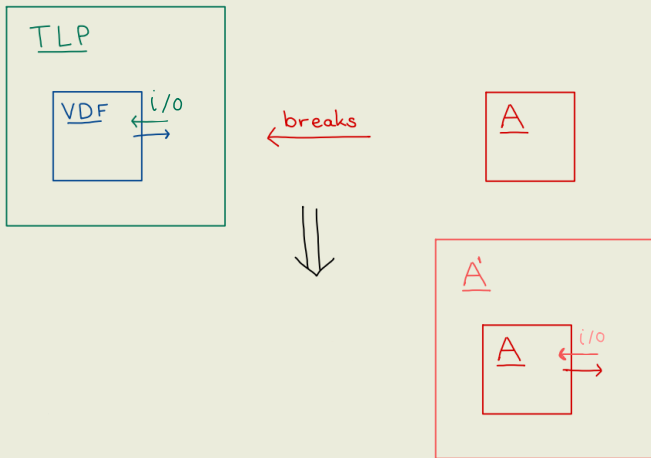
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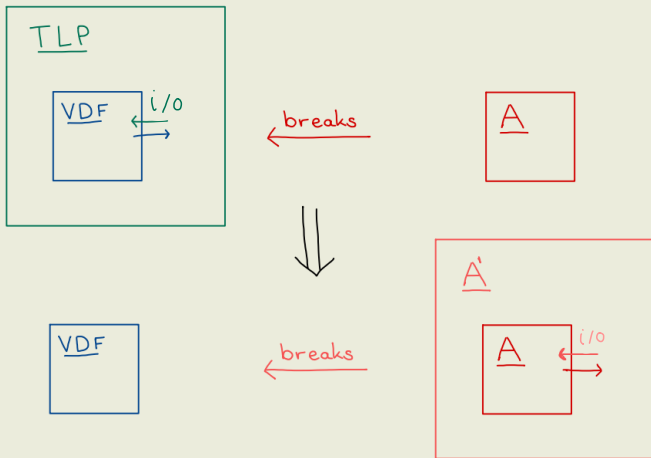
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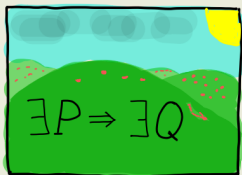
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If  $P$  black-box implies  $Q$ , then in every “world” in which  $P$  exists,  $Q$  exists.

$$P \xrightarrow{\blacksquare\text{-}\text{imply}} Q \quad \Longrightarrow$$



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If there is a “world” in which  $P$  exists and  $Q$  does not exist,  $P$  does not black-box imply  $Q$

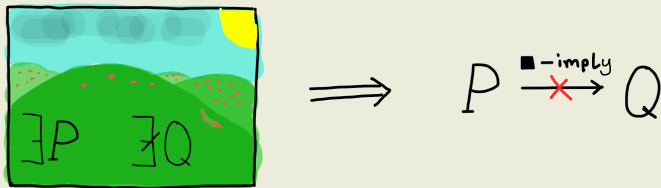


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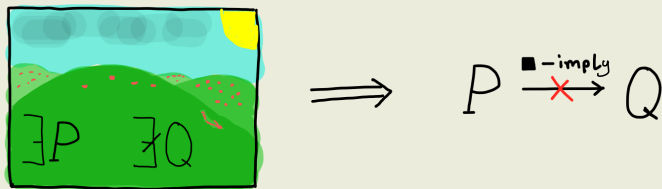
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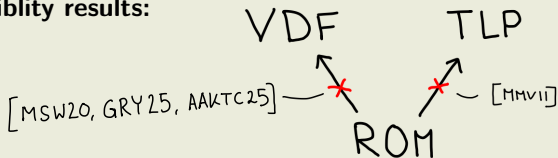
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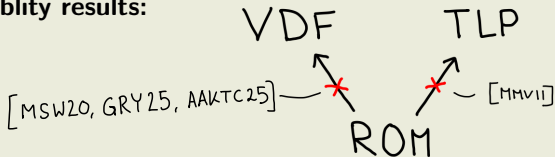


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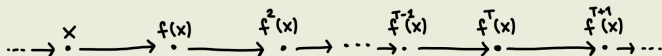
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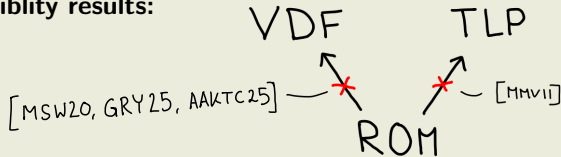


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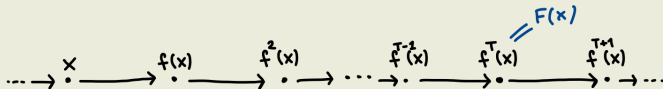
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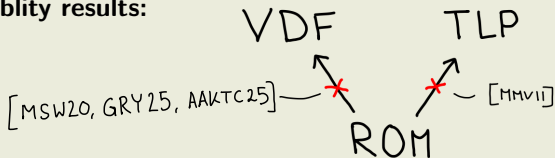


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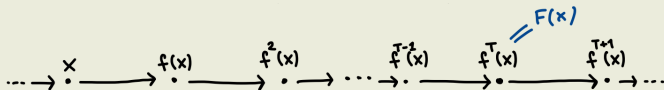
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**Idea:** Extend the ROM with minimal additional oracles s.t. we can construct one of the two primitives, but (still) rule out the other.

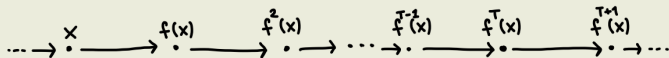
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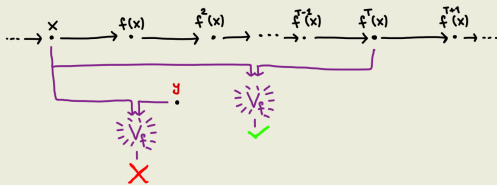
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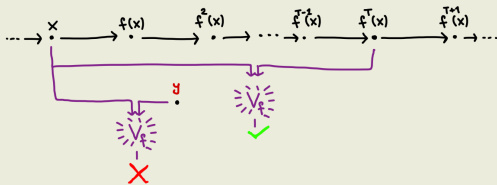
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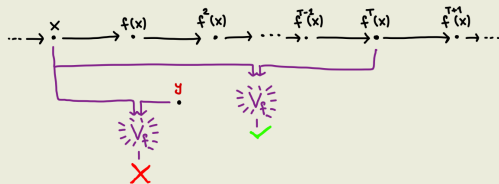
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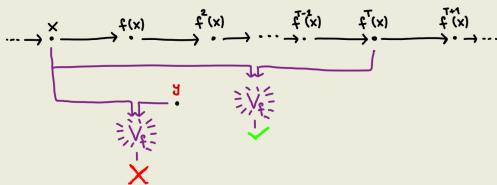


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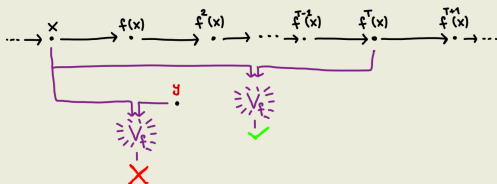


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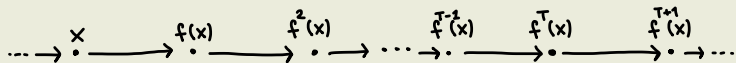
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  - **Key Insight:** We show that given a TLP in this model we can construct a TLP in the plain ROM by simulating the verification oracle  $V_f$  using some extra queries.

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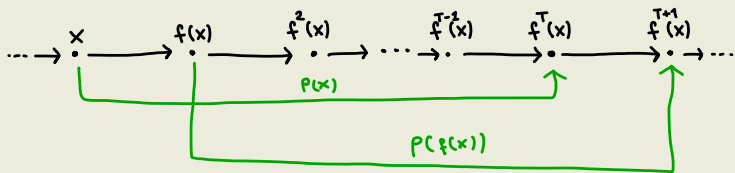
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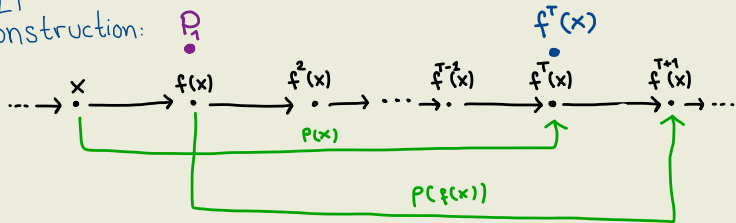


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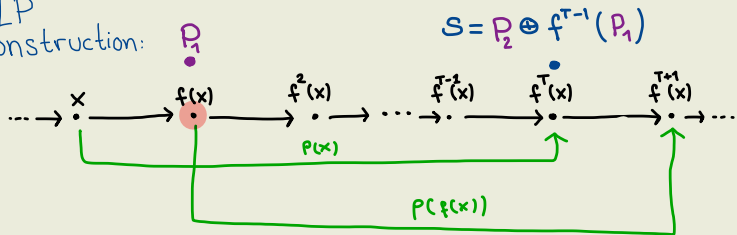
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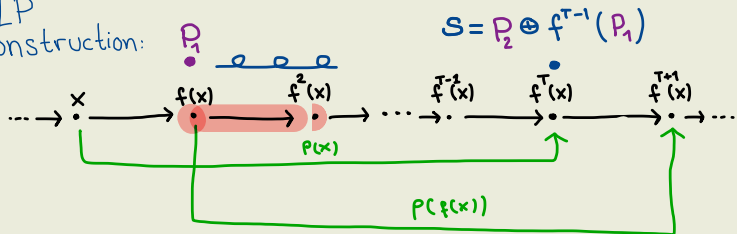
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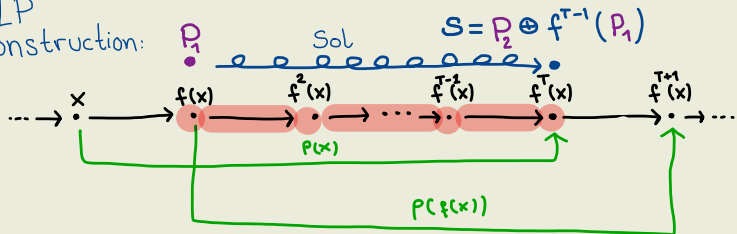
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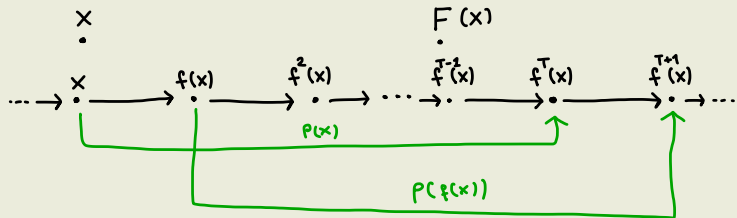
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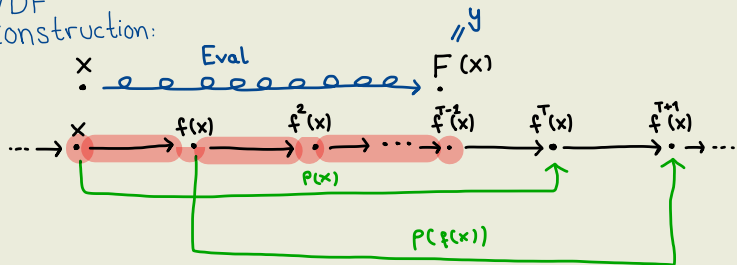
- TLPs exist.
- But: VDFs exist as well!
  - given  $x \in \{0, 1\}^\lambda$ ,  $\text{Eval}(x)$  computes  $F(x) := f^{T-1}(x)$  (no proof)
  - given  $x, y$ ,  $\text{Verify}$  checks if  $f(y) = \rho(x) = f^T(x)$ .

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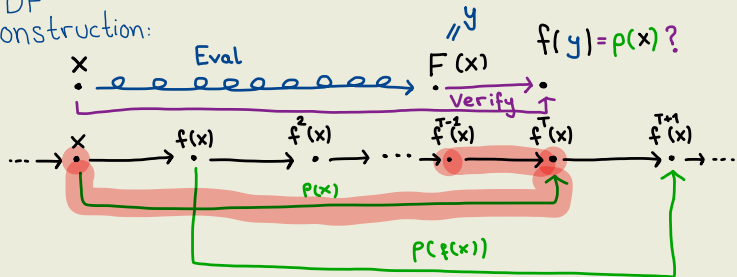
- TLPs exist.
- But: VDFs exist as well!
  - given  $x \in \{0, 1\}^\lambda$ ,  $\text{Eval}(x)$  computes  $F(x) := f^{T-1}(x)$  (no proof)
  - given  $x, y$ , Verify checks if  $f(y) = \rho(x) = f^T(x)$ .

## Construction of $\mathcal{M}_{\text{TLP}}$

Want TLPs to exist, but VDFs to (provably) not exist.

**Naive Idea:** Extend the ROM with “puzzle oracle”:  $\rho(x) := f^T(x)$ .

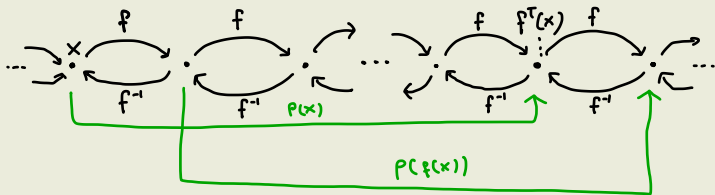
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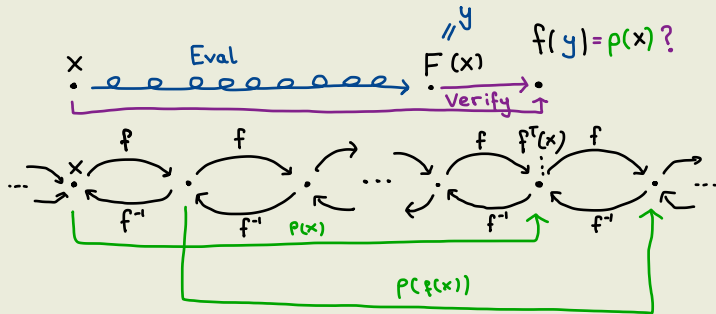
## Construction of $\mathcal{M}_{\text{TLP}}$

**Better Idea:** Let  $f$  be a random permutation, and provide access to  $f^{-1}$ !



## Construction of $\mathcal{M}_{\text{TLP}}$

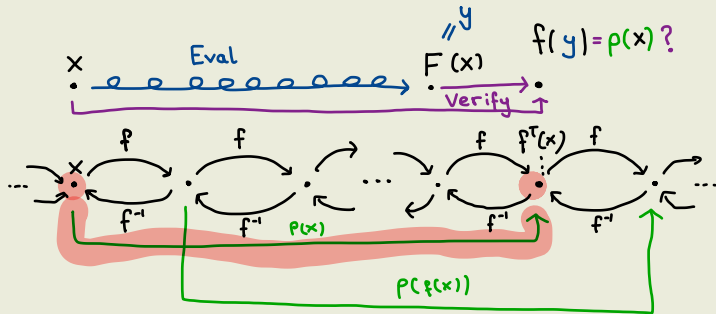
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- Now: the construction of VDF as above is insecure:  $F(x)$  can be computed in 2 steps.

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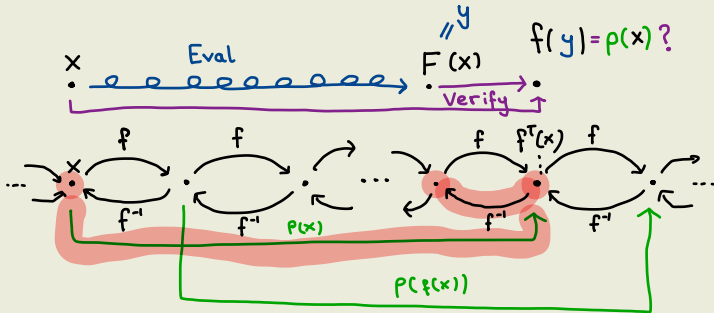
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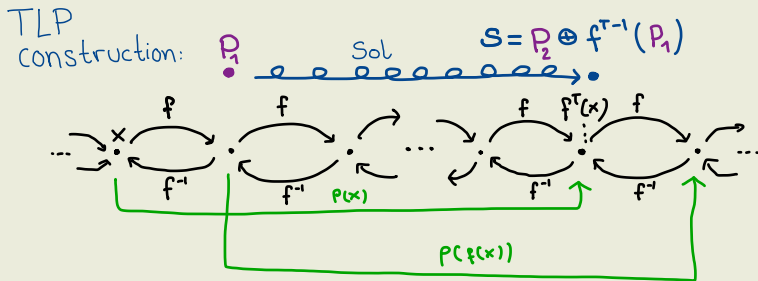


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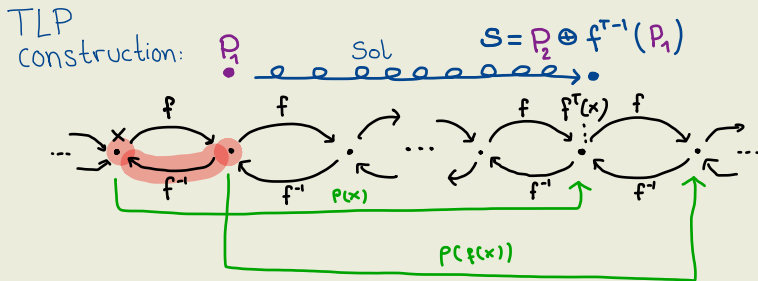
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- Now: the construction of VDF as above is insecure:  $F(x)$  can be computed in 2 steps.
- But: the construction of TLP as above is also insecure.

# Construction of $\mathcal{M}_{\text{TLP}}$

**Better Idea:** Let  $f$  be a random permutation, and provide access to  $f^{-1}$ !

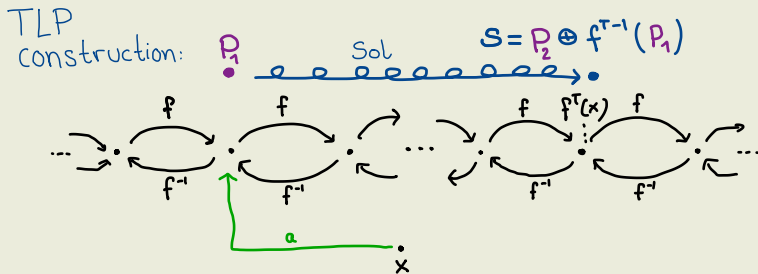


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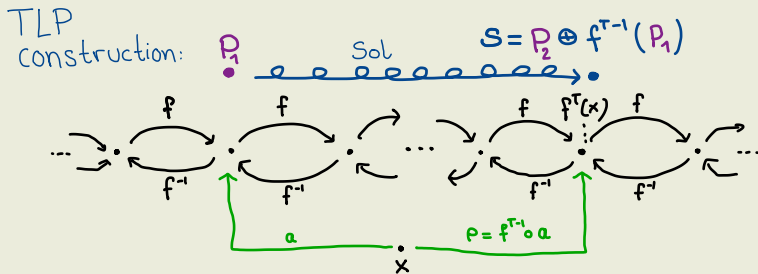
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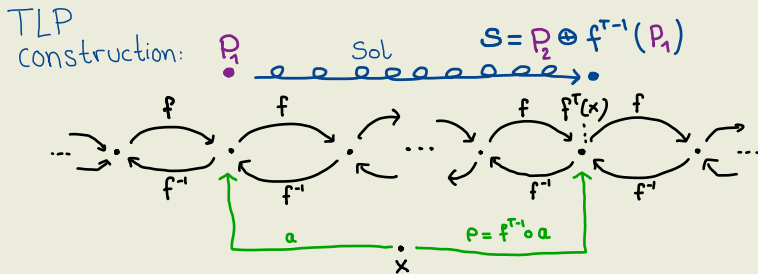
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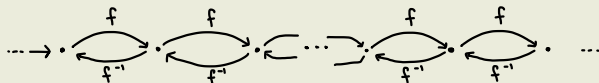


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# Construction of $\mathcal{M}_{\text{TLP}}$

## Final Construction:

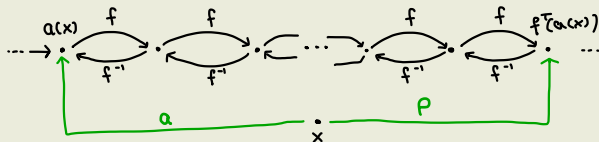
- $f: \{0, 1\}^\lambda \rightarrow \{0, 1\}^\lambda$  is a random permutation
- In  $\mathcal{M}_{\text{TLP}}$ , all algorithms have shared access to:
  - $f$  and its inverse  $f^{-1}$ , and additionally



# Construction of $\mathcal{M}_{\text{TLP}}$

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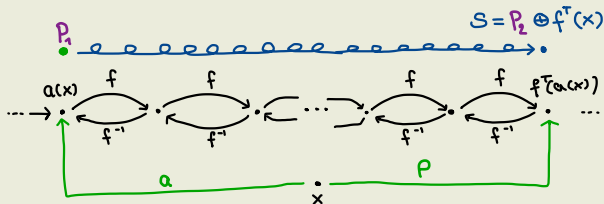
- $f: \{0, 1\}^\lambda \rightarrow \{0, 1\}^\lambda$  is a random permutation
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  - the maps  $x \mapsto a(x)$  and  $x \mapsto \rho(x) := f^T(a(x))$  (“puzzle oracles”)



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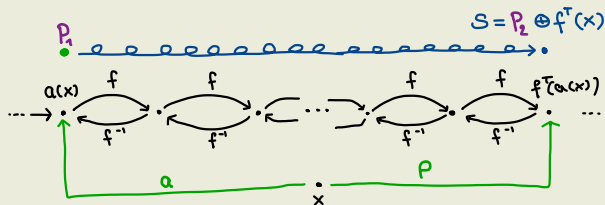
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## Now:

- TLPs exist:  $\text{PGen}(s, T)$  outputs  $(a(x), \rho(x) \oplus s)$  for  $x \leftarrow \{0, 1\}^\lambda$ .
- VDFs don't exist: the proof of [AAKTC25] can be made work in  $\mathcal{M}_{\text{TLP}}$ . (with lots of work)

Thank you!

Questions?

# References I

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